

6 The Horizontal and Vertical Characteristics of the Federal Individual Income Tax, 1966-1977

Marcus C. Berliant and Robert P. Strauss

6.1 Introduction

The purpose of this paper is twofold: to compare and contrast traditional and recent theoretical constructs of horizontal and vertical equity through the use of a general, theoretical framework; and to measure the horizontal and vertical equity of the federal individual income tax,¹ over a significant period of time, through the use of large, microdata files of federal individual income tax returns, and through the use of certain summary of index numbers developed earlier by the authors and based on Wertz (1975).²

In terms of our major theoretical results, we find that the traditional principle of equity, taken to mean "equal treatment of equals," is logically separate from the more recent notion of horizontal equity which suggests

Marcus C. Berliant is assistant professor of economics at the University of Rochester. Robert P. Strauss is professor of economics and public policy at Carnegie-Mellon University.

Initial funding for this research was provided by the Office of Research and Statistics, Social Security Administration, under grant 10-P-98082-3-01. The authors would like to thank Martin David, Kenneth Wertz, Fritz Scheuren, and Carsten Kowalczyk for comments. They also wish to acknowledge the forbearance and general assistance provided by the Carnegie-Mellon Computation Center where all computer work was performed. Responsibility for errors rests with the authors.

1. Throughout this study we examine the ratio of net taxes to measured economic income and interpret this ratio to reflect the equity of the tax system. Much of this chapter addresses the issue of what *equity* may be defined to mean. These measures are *ex post* measures of the relationship between individual taxes and their pretax income. It is therefore unnecessary to account separately for behavioral responses of taxpayers to tax rules that lead them to rearrange their sources of income and ultimately affect their taxes as well. Because we are examining various *ex post* measures over time, we are able to see if stability exists in the observed pattern of vertical and horizontal equity in the system. A disadvantage with examining just one year of data is that the observed *ex post* distribution may reflect transitory reactions to a particular event.

2. See Berliant and Strauss 1983.

that the relative positions of individuals' before-and after-tax income be maintained for horizontal equity to be achieved. While several authors have stated that the classical criteria of equals-treated-equally implies this no-rank-reversal criterion,³ we demonstrate through two simple counter-examples that this is not true. Also, we suggest that the analysis of a tax system's equity is inherently a two-variable problem (the economic position of taxpayers without regard to the tax system, and the taxpayers' effective tax rates), rather than a single variable problem (the distribution of before- or after-tax income).

Generally, our framework permits the distinction between measures of income inequality, and vertical and horizontal equity. The new notion of horizontal equity that requires maintenance of relative rank position may be viewed in this framework, according to our nomenclature, to be a vertical rather than a horizontal equity concept, while inequality measures are found to be income distribution concepts.

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In terms of our major empirical results, we find a number of interesting regularities in the pattern of horizontal and vertical equity of the U.S. personal income tax. Over the period 1966-77 we find that the overall vertical progressivity of the federal personal income tax has remained at a high level—that is, comparisons of pairs of taxpayers in each of the twelve years suggests that at least 80 percent of the comparisons are progressive; that is, those with higher incomes experienced higher effective tax rates than those with lower incomes. By contrast, there is substantial evidence of horizontal inequity. Those taxpayers classified as being in the same economic position were found in 80 percent of the comparisons to experience different effective tax rates; we interpret this to be evidence of horizontal inequity.

While the level of progressivity was generally high, evidence indicates that it has declined somewhat over the sample period (1966-77). Also, we find that the progressivity of the tax system for single taxpayers and married-filing-jointly taxpayers has been declining over the study period. We do not, by contrast, find significant trends in horizontal equity over time for any subgroups.

If we characterize the impact of taxation through the use of the Gini coefficient of after-tax income, an income inequality measure, we find that it is declining over time in a significant fashion generally and for single and married-filing-jointly taxpayers. Thus, the Gini coefficient tells us that the distribution of after-tax income became more egalitarian or equal, while the vertical and horizontal index numbers indicate that a more complex process has been at work, since there has been a decline in progressive components in the system and in increase in regressive components in the tax system over the period 1966-77.

3. For example, Feldstein 1976, Atkinson 1980, Plotnick 1981, and King 1983.

If we examine the overall level of progressivity and horizontal equity by type of filing unit, we find that there are much greater differences among these strata, in the extent to which the tax system creates horizontal inequity, in comparison to the differences in the overall level of progressivity. That is, the tax system tends to be progressive at the same rate, but fails to achieve horizontal equity at the same rate for different types of filing units. Generally, horizontal equity tends to be greatest for single taxpayers and smallest for married-filing-jointly taxpayers. This appears to be related to the high degree of itemization among married taxpayers filing jointly.

The body of this chapter is organized as follows: section 6.2 provides a general conceptual framework within which various equity concepts may be analyzed, and provides a comparison of traditional concepts and measures with others in the literature. The intuition behind the index numbers developed by the authors is also discussed. Section 6.3 describes the microdata files used in the empirical section of the paper, and compares the empirical measures of income with notions of theoretically desired, economic income. Section 6.3 also discusses a number of technical, related issues of how one implements the index numbers developed in section 6.2. Section 6.4 provides the empirical results for our measures of horizontal and vertical equity along with those found in the literature. Section 6.5 concludes.

6.2 Concepts of Horizontal and Vertical Equity

6.2.1 A Framework for Analyzing Alternative Concepts of Equity

We provide here a discussion of alternative horizontal and vertical equity concepts and a rationale for the use of our index numbers, which are relatively novel. Since the emphasis in this paper is primarily empirical, we omit formal proofs of the central propositions here; a more complete study is Berliant and Strauss (1984), where proofs of the propositions stated below may be found.

Summary measures of income and other distributions have long interested economists and statisticians. In a number of related papers Atkinson (1970), Blackorby and Donaldson (1976), Sen (1973), Kondor (1975), Rosen (1978), Fields and Fei (1978), and King (1983) have pointed out that index measures of the income distribution should be consistent with a social welfare function. Atkinson (1970), for example, develops on the basis of certain characteristics, or postulates concerning an underlying social welfare function, a particular index of vertical income inequality, while Fields and Fei (1978) examine a number of commonly used index measures (coefficient of variation, Gini coefficient, Atkinson's index, and



Theil's index) to see if they are consistent with three axioms that they recommend for vertical measures of income inequality.⁴

Related to the broad area of income distribution has been a literature in public finance concerned with the measurement of the progressivity of a tax system. For example, Musgrave and Thin (1948) examined a variety of formulas for calculating the degree of progression of a personal income tax system. Much earlier, Mill (1921) sought to ascertain whether one could produce a progressive income tax regime if one knew consumers' marginal utilities of income; Samuelson (1947) made this approach more precise.

Most recently, Feldstein (1976), Atkinson (1980), and Plotnick (1981) have rekindled interest in horizontal equity. In an important recent paper, King (1983) unified consideration of the vertical and horizontal characteristics of tax systems by using a social welfare function approach suggested by these earlier papers.

In this recent literature, the term *vertical equity* refers to any comparison of the after-tax income distributions generated by tax systems. Measures of vertical equity (or inequity) are essentially measures of after-tax income inequality. The term horizontal equity in this literature refers to the measurement of any characteristic of a tax system that requires the use of the prior- or pretax positions of taxpayers. For example, a measure of horizontal equity or inequity might require the use of the pretax income of each consumer.

It is possible to construct a general framework that incorporates this scheme as well as others. A set of pretax attributes is postulated to be a vector space of variables such as location, income, and marital status parameterized in Euclidean space. There is a vector of pretax attributes associated with each consumer. If a tax system is defined to be a map from any vector of pretax attributes of a consumer to after-tax income, then it is impossible to separate the ranking of tax systems from the distribution of pretax attributes. This is due to the idea that a tax system that has an inequitable feature that applies to no consumer should not be ranked differently from a tax system without this feature. Hence, an equity concept (of any type) is defined to be any ordering over the product of tax systems and attribute distributions; a tax system-attribute distribution pair is our basic construct. The measures of vertical and horizontal equity discussed above are all equity concepts in the sense just defined.

In the literature described above, a restriction placed on horizontal equity concepts is that there should be no rank reversals in moving from pretax to posttax income. In other words, if a tax system-attribute distribution pair, (reflected, for example, by an effective tax rate and before-tax

4. Whether or not such index numbers indeed have all the desirable properties of their parent social welfare functions is discussed by Berliant and Strauss 1983. It should also be noted in this regard that if the operational measure of equity is a multivariate index number, it generally cannot be uniquely deduced from a social welfare function.

income pair of values) satisfies a no-rank-reversal condition, then this tax system-attribute distribution pair ought to be placed in the highest equivalence class of the ordering associated with a horizontal equity concept (or the *lowest* equivalence class of the ordering associated with a horizontal inequity concept). We call the highest and lowest equivalence classes of an ordering the extreme equivalence classes generated by that ordering.

Is the no-rank-reversal condition sufficient, necessary, or both, for placement of a tax system-attribute distribution pair in an extreme equivalence class? It can be easily shown that such a condition is in fact only sufficient for placement in an extreme equivalence class. For example, this condition is only sufficient for the measures in King (1983). More generally, any condition that is postulated to be sufficient to assign a tax system-attribute distribution pair satisfying the condition to an extreme equivalence class of an equity concept is called an *equity principle*. An example of an equity principle is the no-rank-reversal condition. It is obvious that some equity principles are stronger than others and that the weaker equity principles have larger extreme equivalence classes.

The more traditional scheme that we employ below differs substantially from the horizontal-vertical scheme used by the authors listed above. The traditional ideas about equity with which we are concerned seem to divide equity measures into three categories rather than the two noted above, while at the same time using a similar nomenclature. Indeed, we believe that this has been the source of some confusion. Therefore, we use three terms—income inequality, a concept of horizontal equity that we label HE, and a concept of vertical equity that we label VE—in specific ways which we define below. These three concepts of equity correspond, in our view, to precise definitions of older (or classical) notions of income inequality, and vertical and horizontal equity.

These three categories of equity concepts are used, for example, by Musgrave (1959). By creating a distinction between the distributive and allocative functions of government, Musgrave makes a distinction between income redistribution (a distributive idea), and the determination of the method of taxation for providing public goods (an allocative idea). The latter includes, as a partial solution, the use of taxes based on ability to pay, which in turn includes as considerations vertical and horizontal equity. It is in this sense that we shall develop three equity concept classifications.

The first category of equity concepts that we call *income inequality* is the same as the term *vertical equity* as used by the recent literature; it consists of all equity concepts that are functions of only the after-tax income distribution generated by a tax system-attribute distribution pair.

The second category of equity concepts, *HE*, derives from the view, stated, for example, by Musgrave and Musgrave (1980): "Perhaps the most widely accepted principle of equity in taxation is that people in equal positions should be treated equally."

Of course, this statement is only an equity principle, not an equity concept. It requires that a tax system-attribute distribution pair that treats equals in the same manner be placed in an extreme equivalence class of an equity concept. However, the principle does indicate that one must be able to say who are equals and who are not equals in order to evaluate a tax system-attribute distribution pair. Hence, we divide the space of attributes into cells, where those in each cell are considered to be equals by the policy analyst. This may seem arbitrary, but must be done in order to use the traditional notion of equity, and, from a pragmatic point of view, must be done in order to evaluate any index measure since data are always provided in aggregates.

Once this classification is accomplished, an HE equity concept is an ordering such that if the posttax income distribution for each cell of equals for two tax system-attribute distribution pairs is the same, then the two pairs are equivalent under the ordering. In this way only changes in the comparisons of equals can alter the ranking of a pair. In other words, equals are treated in the same manner by both pairs without regard to how unequals are treated. Examples of such measures can be found in Wertz (1975, 1978) and Berliant and Strauss (1983). Also, Pechman and Okner (1974) study empirically variations in effective tax rates by income class; this is essentially an example of a measure of HE as well.

Our development of the third equity concept, *VE*, is complementary to the concepts of HE and distributional equity presented above. A measure of *VE* is defined to be an equity concept that is neither an HE equity concept nor an income inequality equity concept. That is, measures of *VE* do not depend solely on the posttax income distribution (they depend on some pretax variables), nor do they depend solely on the posttax position of equals. Thus, they involve pre- and posttax positions as well as comparisons of taxpayers who are not equals.

This completes the development of the two schemes for categorizing equity concepts. Note that the second, traditional classification scheme yields a finer, and, in our opinion, more natural partition of equity concepts. One can say more precisely what an index number is measuring when it is classified using the second scheme. The index numbers implemented empirically below to evaluate progressivity and equity are respectively *VE*, and *HE* satisfying the principle of treating equals equally.

Moving now to an examination of the two equity principles used most frequently in the recent literature—those principles dealing with no-rank-reversal and equals-treated-equally—it can be shown using two counterexamples that neither one implies the other. That is, equals-treated-equally is neither necessary nor sufficient for a tax system-attribute distribution pair to satisfy the no-rank-reversal criterion.

For the first counterexample, taxpaying units are evenly divided between two narrow pretax income brackets—one high and one low—where

the brackets have the same width and the same internal distribution within each bracket. The brackets also have substantial space between them without any taxpaying units (see fig. 6.1). Further, suppose the tax-transfer system maintains the overall distribution of these units, but is such that the corresponding units in each band switch places. Certainly, given that these units *within* each band are considered to be equals, this tax system conforms to the classical notion of equity, that of equals being treated equally. However, this tax system also plays havoc with the rank ordering of *all* of the units. Thus, changes in the rank ordering do not imply that there are horizontal inequities present in the tax system.

Two obvious objections may be raised to the structure of this example. First, the term *equals* is never defined; but this is not needed since the bands can be made as narrow as necessary (even degenerate). Second, no real-income distribution looks like this one. However, it is equally obvious that this example may be embedded in a larger distribution while maintaining its purpose and conclusion.

The second counterexample postulates a pretax regime with one narrow income bracket in which the entire population is concentrated (see fig. 6.2). Suppose the tax-transfer system spreads the distribution proportionally over a much wider range (i.e., its support becomes larger). Certainly the rank ordering of all individuals does not change under this tax scheme. Also, if the pretax income band is narrow enough to allow all taxpaying units to be considered equals, then the tax system is not horizontally equitable in the classical or HE sense; some taxpayers receive windfalls while others experience huge losses through imposition of the tax system. Thus, tax systems characterized by horizontal inequities do not necessarily change the rank order of taxpaying units.

These counterexamples have demonstrated that each equity principle must be justified *independent* of the other if one is used as an underlying assumption for the measurement of horizontal equity. Of course, they

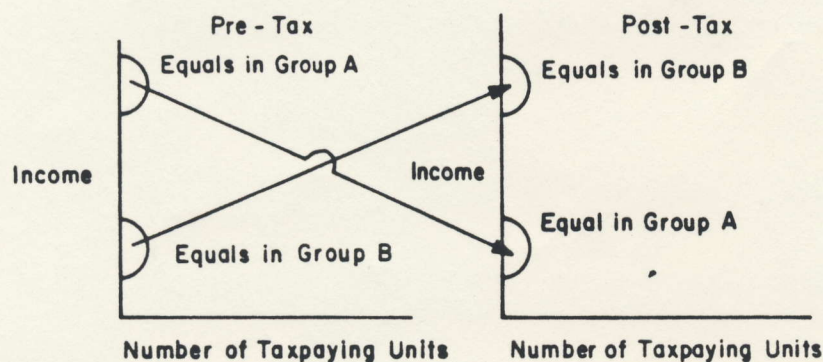


Fig. 6.1.

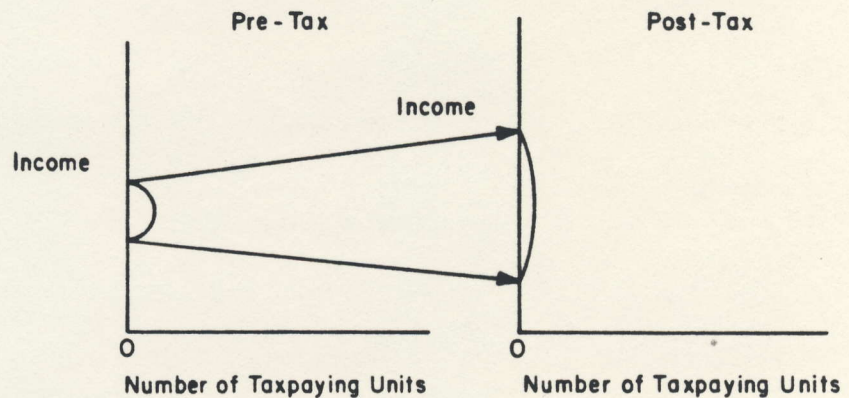


Fig. 6.2.

might also enter as constraints in other models. However, the following result relates the two principles in a different way:

PROPOSITION: If cells of equals are singletons in the space of attributes,⁵ and equals are not treated equally by a tax system–distribution pair, then there exists a ranking of taxpayers so that the tax system reverses some ranks. If a tax system–attribute distribution pair generates a rank reversal, then there exists a set of cells of equals in the space of attributes such that equals are not treated equally.

In summary, we have treated classification schemes for equity concepts or measures of vertical and horizontal equity to clarify some semantic problems and to uncover the assumptions behind various measures. We have also examined the relationships between two commonly used equity principles. To develop a specific measure, one must not only decide on a classification scheme and category along with perhaps an equity principle, but must make other assumptions as well. We have indicated where the measures that we favor fit in; a more complete mathematical development of them may be found in Berliant and Strauss (1984).

To compare a variety of other approaches, a broad selection of index numbers are calculated in the empirical work that follows. Their algebraic statements in consistent notation may be found in Appendix A.

6.2.2 Classifications of Vertical and Horizontal Equity

We now turn to the matter of providing operational criteria that permit us to make distinctions between horizontal and vertical equity in the sense of HE and VE; we provide here the criteria used to classify pairs of attributes. To describe the vertical characteristics of the tax system, we follow Wertz (1978) and partition taxpayers into three parts: the fraction of taxpayers whose tax liability is progressively distributed, ϕ ; the fraction of

5. A singleton is a set consisting of a point.

taxpayers whose tax liability is proportionately distributed, θ ; and, the fraction of taxpayers whose tax liability is regressively distributed, γ . Note that by construction, $\phi + \theta + \gamma = 1.0$. Also, note that the concepts employed are relative concepts obtained by making pairwise comparisons of relative income and effective tax rate positions. A comparison of two taxpayers shows progressivity when both the income and effective tax rate of one taxpayer are greater than the income and tax rate of another taxpayer. Proportionality occurs when the incomes of the two taxpayers being compared are different, but the effective tax rates are the same. Finally, regressivity is said to occur when one taxpayer has a larger income but a lower effective tax rate than the other in the comparison.

To ascertain the extent to which taxes are distributed progressively, proportionately, and regressively, we take into account not only the number of occurrences of each type of comparison, but also the degree of income and tax rate disparities. Our subjective judgment is that it matters when scoring such comparisons whether person A with tax rate of 28 percent and person B with tax rate of 20 percent have similar or very different incomes. Thus, actual measurement involves the weighting of each comparison count by the absolute difference in income of each pair of taxpayers.

Similarly, it would seem to matter whether the tax rates of A and B are similar or very different. If A has an income of \$30,000 and B an income of \$15,000, it would seem important to observe whether the respective tax rates were 28 percent and 20 percent, or 32 percent and 18 percent. The former would seem to be less progressive than the latter comparison. When we account for differences in tax rates, however, we weight by the ratio of tax rates rather than the difference in tax rates. We do this for several reasons. First, using the ratio distinguishes more effectively between a paired comparison of tax rates of 14 percent and 10 percent, and 54 percent and 50 percent. While the differences are both 4 percent, the former pair of tax rates are clearly more disparate. Second, using the ratio of rates deals with proportional comparisons. Recall that if the tax rates in a paired comparison were the same, the difference in rates would yield a weight of zero, while the ratio would yield a weight of one. In the second case the property of the three types of comparisons adding to 1.0 is maintained, whereas under the first weighting scheme, proportional comparisons, because of zero weights, drop out.

It should be noted that our analysis of tax rate/income positions is based on effective rates of taxation and pretax income as classification criteria. Another approach would be to compare individuals in terms of how much income they retain after taxation, or their after-tax income. The two are obviously related. If the effective tax rate is t , then the after-tax-income approach to measuring vertical equity involves making comparisons of the quantity $(1 - t)$ among pairs of taxpayers. The scoring of comparisons in terms of progressivity, regressivity, and proportionality would be the same in both instances, except that progressivity would be

deemed to occur when the fraction of retained or after-tax income declined as income rose. It can be shown,⁶ however, that using the after-tax income approach results in index numbers that are not invariant to scalar multiplication. Because such invariance is generally viewed as a desirable property of index numbers, and the after-tax approach fails to maintain it, we shall use the effective tax rates⁷ calculated as the ratio of net taxes to pretax income.

Horizontal equity in the sense of HE, unlike vertical equity, does not admit of progressive, proportional, or regressive distinctions in our framework, but a disparity in treatment of those in the same position. Accordingly, we shall measure the extent to which effective rates are different—instances of inequity—and instances in which effective rates are the same—instances of equity—for pairs of taxpayers. As with our measure of vertical equity, we shall weight the count of such comparisons by the ratio of effective tax rates, since greater disparities in ratios of tax rates are taken to reflect greater horizontal inequity.

Both the horizontal and vertical measures are obtained by making all possible comparisons among pairs of taxpayers and accumulating the weighted counts of each type of classification. Note that in case of the vertical comparisons, a tax system may be said to have, simultaneously, progressive, regressive, and proportional components. This occurs because the comparisons are relative, and the number of comparisons are numerous; for n individuals, there are $n(n - 1)$ comparisons. Normalization of the accumulation of each of the three possible weighted, vertical counts by the sum of the three components provides a description of the fraction of comparisons that are progressive, proportional, and regressive, and as such, provides a simple index score that can be compared over time for various possible tax schemes. Normalization of the weighted counts of horizontal equity and inequity by their sum provides the same sort of information.⁷ See Appendix A for a presentation, in tabular form, of the algebra of various index numbers implemented below.⁸

6.3 Data, Measurement Considerations, and Other Index Numbers

6.3.1 Data Sources and Limitations

In order to measure repeatedly the distribution of federal personal income taxes, we use the publicly available samples of individual income tax

6. See Berliant and Strauss 1983.

7. A more complete development of the intuition and mathematics of these and related, multiperiod index numbers may be found in Appendix 1 of Berliant and Strauss 1983.

8. The index numbers considered throughout this paper relate *ex post* effective tax rates to pretax economic income. Often it is of interest to compare *ex ante* effective tax rates under different tax regimes. For an analogous set of index numbers that keep track of the relative position of taxpayers under different tax regimes, see Berliant and Strauss 1983.

returns maintained by the National Archives. Each year, the Statistics of Income (SOI) Division of the Internal Revenue Service creates a random, stratified sample of several hundred thousand individual tax returns which are used for the annual publication *Statistics of Income Individual Income Tax Returns*. A sample of this file is typically drawn by the U.S. Treasury Department for revenue-estimating purposes in support of tax legislation. This sample is used in conjunction with the department's microsimulation model of the individual income tax,⁹ and is usually described as the tax model data file.

Also, SOI creates a sample from its large SOI sample and provides it annually to the National Archives. This public-use sample of anonymous individual income tax returns is usually at least twice the size of the sample provided to the Treasury Department, though it is less complete than the Treasury sample in that the Office of Tax Analysis usually synthetically adds additional income information to the sample and creates new weighting schemes to permit the sample to forecast for more recent periods. At the time this project was initiated (1980), annual files for 1966 through 1977 were available from the National Archives and are accordingly the focus of this study.

As is well known, information on the income and tax position of individuals and families is available from a variety of sources; each source has certain strengths and weaknesses. The files used in this study reflect the income and individual income taxes of taxpayers. Other files such as the Current Population Survey (CPS) are much richer sources of demographic information and information about nonwage income. The CPS data base contains the richest information about nonwage income, but does not have actual data on taxes paid. Researchers interested in using this broad definition of income have had to either synthetically match tax information or simulate personal taxes to examine effective tax rates. The SOI data base contains actual tax information, but does not have as broad a definition of income as these other sources. Thus, the SOI does not have information about low-income individuals,* neither are they in the file nor are their sources of income given. Various cash and noncash sources of transfer income are not recorded for federal tax purposes and are thus unavailable to this study. Since variations in effective tax rates over time is the primary subject matter of our research, we have chosen to utilize the richer source of information on taxes actually paid by individuals and sacrificed access to a broader definition of income.

Both the SOI and CPS information fail to reflect nonmarket income captured by the personal income concept in the national income accounts. Personal income as defined in the national income and product accounts,

9. See Wycarver 1978 for a description of the simulation model and techniques used to extrapolate historical data to more current time periods.

is substantially broader than adjusted gross income, total money income, or the concept of economic income we were able to construct from the available data files. Our income concepts do not capture, for example, interest on state and local bonds, which is tax exempt for federal purposes and therefore not reported on tax forms.

Table 6.1 displays the components of economic income available for this study. Such items as wages and salaries, interest and dividend income (return to capital), and types of business income from farming, sole proprietorships, rents, and royalties are contained in our measure, as are such items as long- and short-term capital gains, gains from installment sales, and pension income.

Table 6.1 Components of Economic Income Used in Analysis by Year

Component of Income	66	67	68	69	70	71	72	73	74	75	76	77
Wages and salaries	X	X	X	X	X	X	X	X	X	X	X	X
Interest income	X	X	X	X	X	X	X	X	X	X	X	X
Gross dividends	X	X	X	X	X	X	X	X	X	X	X	X
Interest income	X	X	X	X	X	X	X	X	X	X	X	X
Gross business or profession income	X	X	X	X	X	X	X	X	X	X	X	X
Short-term capital gains	X	X	X	X	X	X	X	X	X	X	X	X
Long-term net capital gains	X	X	X	X	X	X	X	X	X	X	X	X
Farm income	X	X	X	X	X	X	X	X	X	X	X	X
Rental income	X	X	X	X	X	X	X	X	X	X	X	X
Royalty income	X	X	X	X	X	X	X	X	X	X	X	X
Partnership income	X	X	X	X	X	X	X	X	X	X	X	X
Small business corporation income	X	X	X	X	X	X	X	X	X	X	X	X
Estate and trust income	X	X	X	X	X	X	X	X	X	X	X	X
Capital gain distributions							X	X	X	X	X	X
Taxable portion of pensions	X	X				X	X	X	X	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Fully taxable pensions							X	X	X	X	X	X
Gross pensions			X	X	X					X	X	X
Alimony	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	X	X	X	X	X	X	X
State income tax refunds	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	X	X	X	X	X	X	X
Premature distribution from IRAs and Keogh plans	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{2}{2}$	X	$\frac{2}{2}$
Miscellaneous income	X	X	X	X	X	X	X	X	X	X	X	X
Supplemental schedule gains							X	X	X	X	X	X
Ordinary gains	X	X	X	X	X	X						
Other gains	X	X	X	X	X	X						

Sources: SOI files.

Notes: (1) Shown separately but also included in gross pension; (2) included in miscellaneous income.

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Table 6.2 Comparison of Income Concept Used in Study to Adjusted Gross Income and BEA Personal Income Concepts (\$ in billions)

Year	Sample Count ^a	Economic Income ^b	AGI	BEA Pers. Income	Sample Wages	BEA Wages	Sample/BEA Wages %
1966	86,610	482.8	468.5	588.2	379.9	398.4	95.4
1967	87,160	524.4	504.8	629.9	411.3	426.9	96.3
1968	91,484	581.8	555.5	690.6	451.6	469.6	96.2
1969	93,065	623.6	603.2	754.7	497.2	515.7	96.4
1970	95,316	653.5	631.9	811.1	531.9	548.7	96.9
1971	99,137	696.0	672.6	868.4	565.2	581.5	97.2
1972	106,581	775.9	746.8	951.4	621.1	635.2	97.8
1973	112,440	853.4	828.1	1065.2	687.3	702.6	97.8
1974	98,645	924.6	909.9	1168.6	759.9	765.2	99.3
1975	100,851	964.3	947.0	1265.0	794.5	806.3	98.5
1976	164,137	1105.9	1054.6	1391.2	881.0	889.9	99.0
1977	155,212	1173.5	1159.4	1540.4	969.9	983.2	98.6

^aNumber of returns on SOI data file.^bSee text for definition.

Table 6.2 displays the number of (unweighted) observations used by year in the analysis below, along with the total economic income and adjusted gross income (AGI) that were on the annual tapes.¹⁰ Our measure of economic income was typically 1 to 3 percent larger than adjusted gross income each year, though considerably smaller than the personal income measure estimated by the Bureau of Economic Analysis (BEA). A sizeable portion of the difference between either AGI or our measure of economic income and personal income is due to various types of transfer payments. Since many of these transfer payments accrue to nontaxable, low-income individuals and families and do not affect their tax status (they simply are not in the tax system and are not taxable), part of the discrepancy between personal income and our measure of income is not problematical for our purpose. That is, since the purpose of this study is to measure the vertical and horizontal characteristics of the tax system, the fact that some types of income are not in our measure is not problematical to the extent that such income accrues to those outside the tax system.

The last columns in table 6.2 indicate wages and salaries in our data files in comparison to those estimated by BEA. Of interest here is that the coverage ratio is quite high—between 95 and 99 percent. Thus, at least for wage and salary income, our estimates of the vertical and horizontal equity of the tax system should be reliable.

10. It should be noted that our control totals of weighted, adjusted gross income compared favorably with published totals in the pertinent *Statistics of Income* publication or that displayed in table 8.13 of the 1981 supplement to the *Survey of Current Business*, U.S. Department of Commerce 1981.

6.3.2 Other Index Numbers, and Computational and Related Considerations

As noted earlier, there is a substantial index-number literature devoted to ascertaining the structure of income inequality. Since this project involved the repeated computer analysis of better than 1.29 million (anonymous) tax returns,¹¹ we implemented, in addition to the vertical and horizontal measures developed above, seventeen other measures of the vertical distribution of after-tax income, and one additional measure of the horizontal distribution of taxes which we gleaned from the literature. Appendix A provides in a consistent mathematical form these index numbers and the appropriate reference. Of interest is that sometimes various students of index numbers have different definitions of what is purportedly the same index number.¹²

With roughly 100,000 observations per year, calculation of each independent vertical measure, say γ and ϕ , would require 1×10^{10} calculations each per year, which was clearly too burdensome computationally. In order to make the computations tractable, we elected to classify returns into 25 income classes, and 114 tax rates classes.¹³ This dimensionality was used in our earlier study, and thus permits comparison of results from the Treasury and public-use SOI data bases. The finer division of tax rates is justified by our interest in the extent of progressivity in the system. Because we performed the analysis over time, we created income intervals that corresponded to 4 percent of the weighted number of tax returns each year.¹⁴

11. For each year under study, substantial effort was involved in converting and checking, against published tables, the twelve data files from the National Archives. Their files provided to the project were in IBM packed decimal format. We then converted them to Dec ASCII, extracted the relevant variables for the analysis, constructed control totals of AGI and the cumulative distributions in \$500 intervals to properly choose income intervals, and performed the index number calculations per se. On average, each file was passed four times. Even using high-density storage formats, many years required the use of multireel data files.

12. Compare Theil 1967 with Bourguignon 1979, for example.

13. Even this reduction in the size of the computation problem results in many calculations. Using a 25×114 matrix creates 2,850 cells which need to be compared to 2,849 cells or 8.1 million potential comparisons. Of course, many cells are empty (low-income taxpayers do not face high effective rates (and vice versa), so initial identification of nonzero cells can reduce materially the computational burden. Generally, under 1,000 cells needed to be considered. Copies of the algorithms developed for this project are available from the authors upon request.

As is apparent from the mathematics of our vertical and horizontal index numbers, the dimensionality of the income classes and tax rate classes will affect the overall level of results obtained. In our earlier study, Berliant and Strauss 1983, we experimented with widening the tax rate intervals from single percentage points, as implemented in this paper, to intervals of four percentage points. The vertical equity scores remained essentially the same, while horizontal equity levels rose. In particular, this fourfold widening in the tax rate classification was accompanied by a twofold improvement in the measured level of horizontal equity.

14. It is worth noting here that the income intervals we used are rather different than those used routinely over the years by the Treasury Department in their policy analysis. Generally, our income classes are much finer in the lower and middle ranges of the income distri-

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In our earlier study we found that stratifying the analysis by type of filing unit (single, married filing jointly, married filing separately, and head of household) revealed the greatest differences in horizontal and vertical equity, as contrasted with other strata such as those who itemized and those who did not. Accordingly, we stratified our analysis by filing type, and, in 1974, by whether or not a spouse with wage and salary income was present. Unfortunately, limitations of funding for computer resources prevented the complete exploitation of this very rich set of data.¹⁵

6.4 Empirical Results

We present here the empirical results of applying the index numbers developed above, and detailed in Appendix A, to the data for 1966-77, in terms of overall measures of vertical and horizontal equity and stratified by filing status.

6.4.1 Overall Results, 1966-77

Panel A of table 6.3 presents the overall results for all filers and indicates that the extent of overall progressivity in the U.S. personal income tax was high. In 1968, 97.7 percent of the weighted vertical comparisons displayed progressivity. This represents the highest progressivity score recorded over the study period. The lowest progressivity score recorded was in 1966 when only 87.5 percent of the vertical comparisons displayed progressivity. Our vertical results with the public-use SOI data correspond to those obtained with the Treasury tax model sample and reported in Berliant and Strauss (1983). The latter data source recorded vertical progressivity scores of .882 in 1973 and .891 in 1975 while the public data source recorded scores of .890 in 1973 and .871 in 1975. As noted earlier, the Treasury tax model sample contains certain income imputations not available in the public samples, and, in the years in question, contained only 50,000.¹⁶

Since the vast majority of vertical comparisons display progressivity, it is not surprising that relatively small amounts of regressivity and proportionality are observed. Generally, between 8 to 11 percent of the comparisons display regressivity, and between 1 to 2 percent of the comparisons display proportionality.

bution compared to their classifications; the Treasury income groupings tend to focus attention on higher-income taxpayers. For general, statistical purposes, use of the four percent point intervals is the more appropriate methodology.

15. Also due to resource limitations, we have been unable to analyze in a parallel manner the panel of matched personal tax returns jointly provided to the project by the Statistics Division of the Internal Revenue Service, Social Security Administration, and the Office of Tax Analysis, U.S. Treasury Department.

16. Horizontal equity scores are, however, notably different. Those obtained using the Treasury data suggest greater inequity.

age percentage

Table 6.3 Vertical and Horizontal Index Values

A. All Filers							
Year	Prog %	Regr %	Prop %	Equity %	Ineq %	Avg Rate	Gini
1966	0.875	0.101	0.024	0.181	0.819	0.302	0.452
1967	0.877	0.099	0.024	0.170	0.830	0.289	0.457
1968	0.977	0.000	0.023	0.157	0.843	0.310	0.463
1969	0.878	0.102	0.020	0.150	0.850	0.323	0.459
1970	0.877	0.100	0.023	0.159	0.841	0.292	0.441
1971	0.892	0.087	0.021	0.164	0.836	0.310	0.448
1972	0.902	0.077	0.021	0.172	0.828	0.298	0.450
1973	0.890	0.091	0.020	0.166	0.834	0.357	0.457
1974	0.874	0.108	0.018	0.154	0.846	0.415	0.462
1975	0.871	0.108	0.021	0.163	0.837	0.471	0.465
1976	0.901	0.078	0.020	0.211	0.789	0.318	0.455
1977	0.864	0.119	0.017	0.183	0.817	0.498	0.471

B. Head of Household

Year	Prog %	Regr %	Prop %	Equity %	Ineq %	Avg Rate	Gini
1966	0.881	0.096	0.023	0.149	0.851	0.243	0.354
1967	0.854	0.123	0.023	0.126	0.874	0.241	0.332
1968	0.977	0.000	0.023	0.130	0.870	0.238	0.331
1969	0.910	0.069	0.021	0.180	0.820	0.285	0.373
1970	0.875	0.101	0.024	0.167	0.833	0.257	0.348
1971	0.879	0.098	0.023	0.145	0.855	0.312	0.341
1972	0.899	0.077	0.024	0.135	0.865	0.318	0.339
1973	0.889	0.085	0.026	0.140	0.860	0.238	0.355
1974	0.866	0.110	0.023	0.123	0.877	0.329	0.338
1975	0.880	0.087	0.033	0.212	0.788	0.351	0.351
1976	0.891	0.072	0.037	0.332	0.668	0.344	0.366
1977	0.857	0.119	0.024	0.241	0.759	0.413	0.382

C. Married Filing Separately

Year	Prog %	Regr %	Prop %	Equity %	Ineq %	Avg Rate	Gini
1966	0.849	0.115	0.036	0.217	0.783	0.343	0.421
1967	0.843	0.124	0.034	0.204	0.796	0.286	0.416
1968	0.974	0.000	0.026	0.172	0.828	0.284	0.417
1969	0.804	0.177	0.019	0.199	0.801	0.287	0.464
1970	0.859	0.115	0.026	0.169	0.831	0.325	0.407
1971	0.885	0.089	0.026	0.232	0.768	0.261	0.417
1972	0.866	0.105	0.029	0.183	0.817	0.294	0.394
1973	0.863	0.115	0.022	0.200	0.800	0.315	0.420
1974	0.783	0.189	0.028	0.173	0.827	0.349	0.422
1975	0.821	0.151	0.028	0.211	0.789	0.413	0.453
1976	0.872	0.108	0.020	0.212	0.788	0.341	0.405
1977	0.809	0.170	0.021	0.200	0.800	0.438	0.476

Table 6.3 (continued)

D. Married Filing Jointly							
Year	Prog %	Regr %	Prop %	Equity %	Ineq %	Avg Rate	Gini
1966	0.887	0.092	0.021	0.097	0.903	0.253	0.343
1967	0.894	0.085	0.021	0.093	0.907	0.250	0.348
1968	0.980	0.000	0.020	0.087	0.913	0.255	0.353
1969	0.890	0.092	0.018	0.085	0.915	0.271	0.344
1970	0.883	0.096	0.021	0.086	0.914	0.255	0.337
1971	0.884	0.096	0.020	0.090	0.910	0.253	0.346
1972	0.900	0.080	0.019	0.091	0.909	0.250	0.343
1973	0.878	0.104	0.019	0.088	0.912	0.282	0.346
1974	0.862	0.121	0.017	0.082	0.918	0.314	0.354
1975	0.859	0.124	0.017	0.091	0.909	0.369	0.360
1976	0.901	0.083	0.016	0.106	0.894	0.252	0.349
1977	0.843	0.144	0.013	0.097	0.903	0.382	0.360

E. Single

Year	Prog %	Regr %	Prop %	Equity %	Ineq %	Avg Rate	Gini
1966	0.942	0.038	0.020	0.467	0.533	0.241	0.486
1967	0.937	0.043	0.020	0.465	0.535	0.220	0.497
1968	0.981	0.000	0.019	0.443	0.557	0.261	0.497
1969	0.944	0.041	0.015	0.433	0.567	0.244	0.487
1970	0.934	0.041	0.025	0.522	0.478	0.224	0.476
1971	0.945	0.032	0.023	0.556	0.444	0.258	0.489
1972	0.939	0.037	0.025	0.564	0.436	0.227	0.496
1973	0.934	0.046	0.020	0.516	0.484	0.365	0.482
1974	0.912	0.071	0.017	0.458	0.542	0.437	0.468
1975	0.915	0.063	0.022	0.457	0.543	0.507	0.480
1976	0.926	0.050	0.024	0.530	0.530	0.266	0.470
1977	0.895	0.085	0.020	0.448	0.552	0.511	0.476

Over time there is evidence of a decline in progressivity; the Pearson correlation between progressivity and time is $-.74$ (see table 6.4). There is also a modest corresponding upward drift in the fraction of comparisons displaying proportionately over time.¹⁷ When there are increases in progressivity, they are accompanied by decreases in observed regressivity in the system, and vice versa.¹⁸

While the U.S. tax system displays substantial progressivity over the period 1966-77, it also displays very substantial horizontal inequity. No more than 21 percent of the weighted comparisons of taxpayers in the

17. The simple correlation between time and the fraction of comparisons displaying proportionality is .7067. See table 6.4 for various bivariate correlation coefficients.

18. The simple correlation between progressivity and regressivity over the study period is $-.9901$. Since the VE index numbers have two degrees of freedom, any bivariate correlation among pairs of VE scores is nontautological.

Table 6.4[illegible][illegible]

same economic circumstance demonstrate similar effective tax rates. In 1969, measured horizontal equity was at its low point with only 15 percent of the weighted comparisons of taxpayers in the same economic circumstance demonstrating similar effective tax rates. We may conclude then that the federal personal income tax is both progressive and horizontally inequitable.

In our earlier study we conjectured that increases in vertical progressivity might be accompanied by reductions in horizontal equity. However, examination of the overall pattern of progressivity and horizontal equity fails to reveal any systematic relationship. The correlation between the fractions of observed progressive comparisons and horizontally equitable comparisons is $-.05$.

If we use the weighted coefficient of variation in effective tax rates as our measure of horizontal inequity, then we observe several regularities. Recall that this measure is the (weighted) sum of coefficients of variation in effective tax rates within each income bracket, and thus reflects the relative amount of within-income bracket dispersion in effective tax rates. This measure of horizontal inequity suggests that there has been, over the period 1966–77, between 30 to 50 percent variation in effective tax rates within income classes—a substantial amount of variation. Also, it appears that this variation is increasing over time; the simple correlation between it and time is $.73$. Since 1973 the coefficient of variation exceeded 40 percent in three of the four years under study.

We also display in table 6.3 the Gini coefficient of income inequality. Interestingly, some evidence exists that the equality in after-tax income is increasing over time; the simple correlation between the Gini and time is $-.67$. More intriguing, however, is the relationship between income inequality as captured by the Gini and horizontal inequity as captured by the weighted coefficient of variation in effective tax rates. The simple correlation between the two measures is $-.531$, which is statistically significant at the 95 percent confidence level. This suggests that when the distribution of after-tax income becomes more equal, the increased equality is accompanied by greater horizontal inequity.

6.4.2 Results by Filing Type, 1966–77

The results of the calculated index numbers by filing type are contained in panels B through E of table 6.3. The high levels of progressivity found in panel A, the overall results, are evident for head-of-household, married-filing-separately, married-filing-jointly, and single taxpayers. Among these four types of taxpayers, single taxpayers display the greatest progressivity. Single taxpayers displayed progressivity in better than 91 percent of the weighted comparisons in all but one of the years under study, while none of the other types of taxpayers displayed such progressivity more than twice in the study period.

Not only does the federal tax system achieve its vertical objective most effectively with single taxpayers, it achieves its horizontal equity objective most effectively with them as well. Single taxpayers demonstrated horizontal equity from 43 percent to 56 percent of the comparisons, depending on the year in question. By contrast, married-filing-jointly taxpayers displayed horizontal equity in only 9 to 10 percent of the weighted comparisons. Undoubtedly the absence of significant variation in exemptions for single taxpayers and the fact that the vast majority of single taxpayers do not itemize explain these two results.

Both single and married filing jointly taxpayers display a downward drift in the degree of progressivity in their vertical comparisons over time. The simple correlations between time and the progressivity scores are $-.74$ and $-.53$ respectively.¹⁹ Thus, while there is no apparent overall movement in the extent of progressivity in the tax system, there appears to be a modest downward trend in the cases of single and married-filing-jointly taxpayers.²⁰

6.4.3 Other Filing Strata

In addition to stratifying the analysis by type of tax schedule, we performed analyses for single and itemized returns over the period 1966-72, and for strata of returns in 1974 corresponding to the presence or absence of wage and salary by sex.

Table 6.5 displays our horizontal and vertical measures for itemizers and non-itemizers. Again, we see that progressivity is substantial for both types of filers, perhaps contradicting the notion of some that itemized deductions reduce the progressivity of the system. In two of the seven years for which the analysis was performed, itemized returns actually displayed somewhat greater progressivity. However, substantial differences can be seen in the horizontal equity scores between itemizers and nonitemizers, as might be expected. Generally, equity is apparent in only 7 to 8 percent of the comparisons among taxpayers who itemized during the study period, while comparable figures for nonitemizers are 29 to 37 percent. These results compare favorably with those obtained in our earlier study. As with the earlier overall results, there are no apparent temporal relations for itemizers and nonitemizers, nor is there any apparent relationship between equity and progressivity scores.

Stratification by the presence or absence in wage and salary earnings by sex provides some interesting comparisons (see table 6.6). For example,

19. Detailed tables supporting these findings are available from the authors upon request and are omitted here due to space limitations.

20. If one estimates simple regressions of the natural log of the progressivity score on the natural log of time for single and married-filing-jointly taxpayers from the data in table 6.3, one obtains elasticities of $-.3$ in the case of single taxpayers and $-.28$ in the case of married-filing-jointly taxpayers with t-ratios in excess of 2.7.

Table 6.5 Horizontal and Vertical Scores
for Itemizers and Standard Filers, 1966-72

A. Itemizers					
Year	Prog %	Regr %	Prop %	Equity %	Inequity %
1966	.877	.102	.021	.084	.916
1967	.880	.099	.021	.079	.921
1968	.978	.000	.022	.0706	.924
1969	.868	.114	.019	.074	.926
1970	.862	.115	.023	.072	.928
1971	.874	.105	.021	.075	.925
1972	.889	.092	.019	.074	.926

B. Standard				
Prog %	Regr %	Prop %	Equity %	Inequity %
.879	.090	.032	.316	.684
.876	.091	.032	.311	.689
.969	.000	.031	.301	.699
.895	.079	.026	.311	.689
.884	.082	.034	.366	.634
.894	.076	.030	.316	.684
.903	.068	.029	.293	.707

Table 6.6 Horizontal and Vertical Scores by Number of Wage and Salary
Earners (1974 data)

Filing Unit	Prog %	Regr %	Prop %	Equity %	Inequity %	Gini	Av Rate
Male W&S >0	.899	.082	.018	.181	.819	.452	.292
Female W&S >0	.926	.052	.022	.339	.661	.441	.257
Male, Female W&S = 0	.770	.212	.018	.097	.903	.652	.677
Male, Female W&S >0	.885	.049	.027	.099	.901	.244	.199
TOTAL	.874	.108	.018	.154	.846	.462	.415

when female wage and salary payments are the only earnings present, horizontal equity is much greater than in any other strata. In this case, 34 percent of the comparisons display equity, compared to, for example, the overall figure of 15.4 percent. For returns that contained wage and salary for both men and women, the situation of working couples, we find that progressivity is high at 88.5 percent, and horizontal inequity is also high at 90.1 percent. Here, however, we also find that the after-tax distribution of income is equal as measured by the Gini coefficient. The Gini for working

couples is .244, almost half of the overall Gini of .462. This suggests that working couples found themselves in similar after-tax-income positions, and may reflect that wage rates in 1974 for working couples were comparable when viewed in terms of family units.

The case in which neither male nor female wages and salaries are present displays the least amount of progressivity and equity of the strata examined. These, of course, would be individuals who receive only non-labor income or retirement or capital income. Note too that these taxpayers have the most unequal distribution of after-tax income; their Gini is .652 compared to the overall figure of .462.

6.4.4 Relations among All Index Numbers

As noted earlier, table 6.4 contains the simple correlations among the twenty-three index numbers (and time) investigated for all filers. There is much information that we will not attempt to summarize here; however, several general comments are in order. First, there is a high intercorrelation among the various income inequality measures. For example, the Gini is highly correlated with a wide variety of measures such as the variance in income, the coefficient of income concentration, Atkinson's three measures (his Gini, and his I evaluated at .3 and .7), the measures of the relative mean deviation, and so forth. Thus, while many of these measures have different numerical values, when compared for a moment in time, or across time, they tend to move closely together and in effect contain similar information.

While the inequality measures are generally highly correlated with each other, they are not always correlated with our measures of progressivity or regressivity. Thus, to the extent one wishes to measure VE in the sense used above, some of the income inequality measures can fail to capture VE type effects. Simple correlations between our progressivity measure and Atkinson's I (.7) were only .42, while the analagous correlation with Theil's measure was .85. This is not surprising, of course, since the inequality measures are not expected to capture the VE effects. This suggests, in turn, that if progressivity or regressivity is of interest to the analyst and the VE concept is persuasive, then some form of progressivity measure as we suggest is appropriate to the task, not an inequality measure. Conversely, if one is interested in the extent to which income inequality changes over time or as a result of proposed changes, then our VE measures are inappropriate measures of such effects.

6.5 Conclusion

We have sought in this paper to create a theoretical framework that allows the comparison of traditional and more recent concepts of horizontal and vertical equity, and to characterize empirically the horizontal and

vertical distribution of federal individual income taxes over a significant period of time. Theoretically, we have shown that the recent concept of horizontal equity, which requires that the pre- and posttax ranks of individuals' income positions be unchanged, is logically divorced from the traditional horizontal equity concept, which requires that the tax system impose identical effective tax rates on individuals in the same (pretax) economic position.

Using carefully defined equity concepts and publicly available data for the period 1966-77, we have found what appears to be substantial and continuing evidence of progressivity in the U.S. personal income tax. However, we also have found substantial and continuing horizontal inequity in the federal personal tax system.

Stratification of our empirical analysis by type of tax schedule reveals that single persons experience the greatest progressivity and horizontal equity in the system, while married-filing-jointly taxpayers experience the least amount of horizontal equity in the system.

Examination of a wide variety of measures of after-tax income inequality reveals that they do not often capture the same information as the proposed vertical and horizontal equity measures, although they are highly related to each other in the sense of being highly correlated. This empirical information is consistent with the above theoretical results.

We have not sought in this chapter to "explain" the extent of measured vertical and horizontal equity, partly due to the size of the task and partly because the initial characterization of equity in the federal individual income tax over time seemed to be the proper point of departure. The extent of observed horizontal inequity is worthy of further study, as the observed discrepancies from some sort of norm of "acceptable" levels of horizontal inequity, compared to observed discrepancies between observed levels of progressivity and what is theoretically possible, would appear to be large. That is, observed progressivity appears to be at least 80 percent of what could be attained, whereas observed horizontal equity is only 10 percent of what could be attained. One may argue that the observed horizontal inequities are the peculiarity of our tax system which provides for exemptions and beneficial tax treatment for various types of activity. However, it is remarkable that units with the same economic position, broadly defined, find themselves facing comparable tax rates in only 10 percent of possible comparisons.

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Appendix A

Algebraic Statement of Various Index Numbers

Key to symbols:

I = # of economic income classes

A = # of after-tax income classes

R = # of effective rate classes

N^j = population in economic income class i , rate class J

P_i = population in after-tax income class i

$T_i = Z_i * P_i$ = total income in after-tax income class i

POP = total population

INC = total after-tax income

D_i^j = population in income class i , change in effective rate class J

Q = # of changes in rate classes (difference between old and new effective rates)

$$VSUM = \sum_{i=1}^I \sum_{k=i+1}^I \sum_{J=1}^R \sum_{L=1}^R N_i^J * N_k^L * |Y_i^J - Y_k^L| * MAX(J/L, L/J)$$

$$HSUM = \left[\sum_{i=1}^I \sum_{J=1}^R \sum_{L=J+1}^R N_i^J * N_i^L / J \right] + \left[\frac{1}{2} \sum_{i=1}^I \sum_{J=1}^R N_i^J * (N_i^J - 1) \right]$$

$$DVSUM = \sum_{i=1}^I \sum_{k=i+1}^I \sum_{J=1}^Q \sum_{L=1}^Q D_i^J * D_k^L$$

Kolm	KOLM	$1000 * \log \left(\sum_{i=1}^A \exp \left(\frac{Z_i}{AVINC} - 1 \right) * \frac{P_i}{1000} \right)$	Kolm 1976
Relative mean deviation #1	RMD1	$\frac{1}{POP} \sum_{i=1}^R \left \frac{Z_i}{AVINC} - 1 \right * P_i$	Atkinson 1970
Relative mean deviation #2	RMD2	RMD1/2	Kondor 1975
Theil #1	THEIL1	$X_i = Z_i / INC$ $\sum_{i=1}^A P_i * X_i * \log(X_i)$	Bourguignon 1979
Theil #2	THEIL2	$\sum_{i=1}^A P_i * Z_i * \log(POP * Z_i)$	Fields and Fei 1978; Theil 1967
Theil #3	THEIL3	$\frac{1}{POP} \sum_{i=1}^A \text{Sign}(Z_i) * P_i * \log(Z_i)$	Theil 1967*
Standard deviation of logarithms	SDL	$\frac{1}{POP} \sum_{i=1}^A P_i * (\log(Z_i / AVINC))^2$	Atkinson 1970
Logarithmic variance	LV	$\frac{1}{POP} \sum_{i=1}^A (\text{Sign}(Z_i) * \log(Z_i) - THEIL3)^2 * P_i$	Kondor 1975
Kuznets ratio	XKUZ	$\sum_{i=1}^A P_i \left \frac{Z_i}{INC} - \frac{1}{POP} \right = RMD1$	Fields and Fei 1978
Average coefficient of variation of effective rates	HIND	$\frac{1}{POP} \sum_{i=1}^I \left\{ \left[\frac{\sum_{j=1}^R N_j^2}{\sum_{j=1}^R N_j * J} \right] * \left[\frac{\sum_{j=1}^R \sum_{k=j+1}^R (J-K)^2 * N_j^k * N_k^j}{\sum_{j=1}^R \sum_{k=j+1}^R (N_j^k * N_k^j) + \frac{1}{2} \sum_{j=1}^R N_j^j * (N_j^j - 1)} \right]^{1/2} \right\}$	

Table 6.A.1 Algebraic Statements for Alternative Vertical and Horizontal Index Numbers

Index Number	Variable Name	Expression	Reference
Progressive (%)	PROG	$\frac{1}{VSUM} \sum_{j=1}^R \sum_{l=j+1}^R \sum_{i=1}^I \sum_{k=i+1}^I N_i^* N_k^* Y_l^* - Y_k^* * L/J$	Berliant and Strauss 1983
Regressive (%)	REGR	$\frac{1}{VSUM} \sum_{j=1}^R \sum_{l=1}^{j-1} \sum_{i=1}^I \sum_{k=i+1}^I N_i^* N_k^* Y_l^* - Y_k^* * L/J$	"
Proportional (%)	PROP	$\frac{1}{VSUM} \sum_{j=1}^R \sum_{l=1}^I \sum_{k=l+1}^I N_l^* N_k^* Y_l^* - Y_k^* $	"
Equity (%)	HGO	$\frac{1}{2*HSUM} \sum_{i=1}^I \sum_{j=i+1}^I N_i^* (N_j^* - 1)$	"
Dispersion	HBAD	$\frac{1}{HSUM} \sum_{j=1}^R \sum_{l=j+1}^R \sum_{i=1}^I N_i^* N_l^* * L/J$	"
Average after-tax income	AVINC	INC/POP	
Variance	VAR	$\frac{1}{POP} \sum_{i=1}^A (Z_i - AVINC)^2 * P_i$	Kondor 1975
Coefficient of variation	CO	$\sqrt{VAR}/AVINC$	Atkinson 1970; Fields and Fei 1978
Mean difference	MD	$\frac{1}{POP^2} \sum_{i=1}^A \sum_{j=i+1}^{i-1} P_i * P_j * Z_i - Z_j $	Kendall 1947
Gini coefficient	GINI	MD/AVINC	Pyatt 1976
Atkinson Gini	AG	GINI/2	Atkinson 1970
Coefficient of concentration	COCON	$\frac{1}{AVINC * POP * (POP - 1)} \sum_{i=1}^A \sum_{j=i+1}^{i-1} Z_i - Z_j * P_i * P_j$	Kondor 1975
Atkinson	AT1, AT2	$1 - \left(\sum_{i=1}^A \left(\frac{1}{AVINC} \right)^{1-\epsilon} \frac{1}{POP} \right)^{\frac{1}{1-\epsilon}}$	Atkinson 1970 AT1: $\epsilon = .3$ AT2: $\epsilon = .7$

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Table 6.A.2 Appendix B Other Index Numbers

Panel A. All Filers									
YR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
66	6060.	0.3211E+08	0.935	2741.	0.452	0.226	0.452	0.108	0.261
67	6421.	0.3759E+08	0.955	2932.	0.457	0.228	0.457	0.109	0.264
68	6822.	0.4444E+08	0.977	3161.	0.463	0.232	0.463	0.113	0.271
69	7056.	0.4245E+08	0.923	3241.	0.459	0.230	0.459	0.109	0.266
70	7643.	0.4476E+08	0.875	3374.	0.441	0.221	0.441	0.100	0.247
71	8166.	0.5523E+08	0.910	3657.	0.448	0.224	0.448	0.104	0.254
72	8758.	0.6278E+08	0.905	3941.	0.450	0.225	0.450	0.105	0.255
73	9195.	0.7252E+08	0.926	4198.	0.457	0.228	0.457	0.108	0.265
74	9549.	0.1017E+09	1.056	4415.	0.462	0.231	0.462	0.111	0.266
75	10170.	0.8626E+08	0.913	4725.	0.465	0.232	0.465	0.111	0.278
76	11340.	0.1046E+09	0.902	5155.	0.455	0.227	0.455	0.105	0.257
77	11650.	0.1157E+09	0.923	5485.	0.471	0.235	0.471	0.112	0.284

Panel B. Head of Household									
YR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
66	5222.	0.1922E+08	0.840	1847.	0.354	0.177	0.354	0.069	0.163
67	5381.	0.1851E+08	0.799	1788.	0.332	0.166	0.332	0.062	0.142
68	5681.	0.2061E+08	0.799	1881.	0.331	0.166	0.331	0.062	0.143
69	5347.	0.2157E+08	0.869	1996.	0.373	0.187	0.373	0.076	0.180
70	5567.	0.1779E+08	0.758	1937.	0.348	0.174	0.348	0.066	0.164
71	5962.	0.2239E+08	0.794	2036.	0.341	0.171	0.341	0.065	0.157
72	6519.	0.2506E+08	0.768	2211.	0.339	0.170	0.339	0.063	0.151
73	6830.	0.3408E+08	0.855	2423.	0.355	0.177	0.355	0.071	0.170
74	7121.	0.3667E+08	0.850	2408.	0.338	0.169	0.338	0.062	0.148
75	7741.	0.3427E+08	0.756	2715.	0.351	0.175	0.351	0.066	0.164
76	8082.	0.4235E+08	0.805	2957.	0.366	0.183	0.366	0.072	0.178
77	8392.	0.4610E+08	0.809	3205.	0.382	0.191	0.382	0.079	0.201

Panel C. Married Filing Separately									
YR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
66	3331.	0.1125E+08	1.007	1401.	0.421	0.210	0.421	0.093	0.221
67	3584.	0.1354E+08	1.027	1492.	0.416	0.208	0.416	0.093	0.225
68	3784.	0.2037E+08	1.193	1580.	0.417	0.209	0.417	0.098	0.224
69	4041.	0.1837E+08	1.061	1874.	0.464	0.232	0.464	0.105	0.253
70	4445.	0.1791E+08	0.952	1807.	0.407	0.203	0.407	0.087	0.210
71	4770.	0.2155E+08	0.973	1988.	0.417	0.208	0.417	0.090	0.212
72	5099.	0.1944E+08	0.865	2012.	0.394	0.197	0.394	0.081	0.194
73	5326.	0.2647E+08	0.966	2234.	0.420	0.210	0.420	0.094	0.230
74	5563.	0.4450E+08	1.199	2350.	0.422	0.211	0.422	0.090	0.223
75	5873.	0.3467E+08	1.003	2662.	0.453	0.227	0.453	0.103	0.265
76	6891.	0.3746E+08	0.888	2789.	0.405	0.202	0.405	0.083	0.207

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Panel A. All Filers

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.6071E+04	0.639	0.320	-17.880	0.1154E+14	8.201	1.231	2.670	0.302
0.7311E+04	0.643	0.321	-17.910	0.1252E+14	8.255	1.247	2.670	0.289
0.7639E+04	0.654	0.327	-17.910	0.1374E+14	8.301	1.294	2.689	0.310
0.1041E+05	0.652	0.326	-18.030	0.1464E+14	8.340	1.294	2.819	0.323
0.1123E+05	0.625	0.312	-18.030	0.1555E+14	8.459	1.155	2.745	0.292
0.1200E+05	0.633	0.317	-18.020	0.1673E+14	8.507	1.180	2.915	0.310
0.1364E+05	0.638	0.319	-18.020	0.1875E+14	8.581	1.257	2.619	0.298
0.1279E+05	0.651	0.325	-18.050	0.2054E+14	8.601	1.288	2.966	0.357
0.1477E+05	0.659	0.330	-18.180	0.2212E+14	8.636	1.016	3.547	0.415
0.1482E+05	0.665	0.332	-18.170	0.2326E+14	8.643	1.314	4.030	0.471
0.1984E+05	0.650	0.325	-18.156	0.2682E+14	8.827	1.227	2.989	0.318
0.2058E+05	0.673	0.337	-18.350	0.2826E+14	8.754	1.310	4.667	0.498

Panel B. Head of Household

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.4391E+04	0.482	0.241	-14.490	0.2679E+12	8.266	0.478	1.879	0.243
0.3814E+04	0.454	0.227	-14.510	0.2970E+12	8.351	0.438	1.160	0.241
0.3673E+04	0.456	0.228	-14.640	0.3543E+12	8.394	0.410	1.455	0.238
0.4460E+04	0.514	0.257	-14.790	0.3897E+12	8.257	0.610	1.903	0.285
0.4456E+04	0.483	0.242	-15.090	0.5210E+12	8.311	0.535	2.242	0.257
0.4281E+04	0.473	0.236	-15.050	0.5615E+12	8.401	0.479	1.960	0.312
0.6712E+04	0.471	0.235	-15.180	0.7018E+12	8.514	0.549	1.450	0.318
0.5196E+04	0.490	0.245	-15.220	0.7932E+12	8.508	0.554	2.136	0.238
0.7352E+04	0.477	0.238	-15.350	0.8883E+12	8.599	0.418	1.873	0.329
0.7772E+04	0.490	0.245	-15.320	0.9610E+12	8.645	0.576	2.051	0.351
0.1253E+05	0.505	0.253	-15.455	0.1096E+13	8.659	0.707	2.214	0.344
0.1171E+05	0.529	0.265	-15.610	0.1252E+13	8.618	0.755	3.452	0.413

Panel C. Married Filing Separately

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.3328E+04	0.591	0.295	-14.850	0.2286E+12	7.699	0.821	2.449	0.343
0.4055E+04	0.576	0.288	-14.820	0.2209E+12	7.749	0.815	2.984	0.286
0.2813E+04	0.582	0.291	-14.660	0.2467E+12	7.830	0.796	2.227	0.284
0.7335E+04	0.648	0.324	-15.050	0.2671E+12	7.805	0.897	3.683	0.287
0.4364E+04	0.572	0.286	-14.540	0.2351E+12	8.004	0.744	2.495	0.325
0.9628E+04	0.589	0.294	-14.380	0.2191E+12	8.091	0.891	1.727	0.261
0.6129E+04	0.554	0.277	-14.540	0.2767E+12	8.183	0.722	1.968	0.294
0.7127E+04	0.588	0.294	-14.630	0.3014E+12	8.132	0.930	2.900	0.315
0.1151E+05	0.595	0.298	-14.950	0.2904E+12	8.163	0.631	4.387	0.349
0.1193E+05	0.626	0.313	-14.790	0.2587E+12	8.101	1.039	5.219	0.413
0.1484E+05	0.574	0.287	-14.540	0.3051E+12	8.429	0.815	3.040	0.341

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Table 6.A.2 (continued)

Panel D. Married Filing Jointly									
YR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
66	8339.	0.3575E+08	0.717	2859.	0.343	0.171	0.343	0.064	0.158
67	8847.	0.4221E+08	0.734	3076.	0.348	0.174	0.348	0.066	0.163
68	9483.	0.5065E+08	0.751	3351.	0.353	0.177	0.353	0.068	0.167
69	9842.	0.4663E+08	0.694	3388.	0.344	0.172	0.344	0.062	0.159
70	10430.	0.4861E+08	0.668	3516.	0.337	0.169	0.337	0.060	0.153
71	11070.	0.6086E+08	0.705	3833.	0.346	0.173	0.346	0.064	0.163
72	12140.	0.6937E+08	0.686	4158.	0.343	0.171	0.343	0.062	0.154
73	12950.	0.8204E+08	0.699	4475.	0.346	0.173	0.346	0.064	0.163
74	13590.	0.1285E+09	0.834	4806.	0.354	0.177	0.354	0.070	0.176
75	14320.	0.9955E+08	0.697	5161.	0.360	0.180	0.360	0.069	0.185
76	16007.	0.1190E+09	0.682	5584.	0.349	0.174	0.349	0.063	0.163
77	16780.	0.1361E+09	0.695	6040.	0.360	0.180	0.360	0.069	0.189

Panel E. Single									
YR	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
66	2832.	0.1054E+08	1.146	1375.	0.486	0.243	0.486	0.123	0.283
67	2981.	0.1264E+08	1.193	1480.	0.497	0.248	0.497	0.125	0.283
68	3169.	0.1486E+08	1.216	1575.	0.497	0.248	0.497	0.130	0.292
69	3250.	0.1346E+08	1.129	1584.	0.487	0.244	0.487	0.123	0.281
70	3651.	0.1502E+08	1.062	1739.	0.476	0.238	0.476	0.113	0.264
71	3878.	0.1951E+08	1.139	1897.	0.489	0.245	0.489	0.119	0.274
72	4245.	0.2276E+08	1.124	2104.	0.496	0.248	0.496	0.122	0.285
73	4440.	0.2390E+08	1.101	2140.	0.482	0.241	0.482	0.120	0.279
74	4630.	0.3121E+08	1.207	2167.	0.468	0.234	0.113	0.113	0.256
75	5022.	0.2717E+08	1.038	2411.	0.480	0.240	0.480	0.119	0.286
76	5829.	0.3729E+08	1.048	2737.	0.470	0.235	0.470	0.113	0.265
77	5980.	0.3800E+08	1.031	2847.	0.476	0.238	0.476	0.116	0.281

Notes:

Column	Index Number	Appendix A Definition
Col. (1)	Average income	(AVINC)
Col. (2)	Variance	(VAR)
Col. (3)	Coefficient of variation	(CO)
Col. (4)	Mean difference	(MD)
Col. (5)	Gini	(GINI)
Col. (6)	Atkinson Gini	(AG)
Col. (7)	Coefficient of concentration	(COCON)
Col. (8)	Atkinson I (.3)	(AT1)
Col. (9)	Atkinson I (.7)	(AT2)

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Panel D. Married Filing Jointly

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.9878E+04	0.467	0.233	-17.460	0.8900E+13	8.724	0.506	2.314	0.253
0.9890E+04	0.474	0.237	-17.460	0.9681E+13	8.769	0.507	2.491	0.250
0.1200E+05	0.480	0.240	-17.490	0.1056E+14	8.833	0.529	2.510	0.255
0.1546E+05	0.470	0.235	-17.600	0.1127E+14	8.880	0.515	2.632	0.271
0.1520E+05	0.463	0.232	-17.580	0.1194E+14	8.948	0.490	2.566	0.255
0.1543E+05	0.476	0.238	-17.580	0.1293E+14	8.982	0.538	2.818	0.253
0.1765E+05	0.473	0.237	-17.520	0.1413E+14	9.109	0.534	2.162	0.250
0.1888E+05	0.473	0.236	-17.580	0.1536E+14	9.139	0.549	2.818	0.282
0.2214E+05	0.478	0.239	-17.670	0.1640E+14	9.150	0.514	3.548	0.314
0.2224E+05	0.495	0.248	-17.680	0.1721E+14	9.163	0.670	3.999	0.369
0.2763E+05	0.481	0.241	-17.652	0.1956E+14	9.346	0.557	3.016	0.252
0.3064E+05	0.493	0.247	-17.820	0.2034E+14	9.291	0.667	4.895	0.382

Panel E. Single

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
0.1569E+04	0.705	0.353	-16.760	0.1808E+13	7.426	1.201	2.213	0.241
0.3852E+04	0.720	0.360	-16.900	0.1967E+13	7.487	1.225	2.025	0.220
0.1913E+04	0.720	0.360	-16.780	0.2191E+13	7.528	1.266	2.035	0.261
0.2254E+04	0.709	0.355	-16.860	0.2290E+13	7.571	1.253	2.005	0.244
0.6025E+04	0.688	0.344	-16.970	0.2413E+13	7.716	1.170	2.156	0.224
0.7442E+04	0.702	0.351	-17.000	0.2553E+13	7.756	1.180	2.349	0.258
0.8952E+04	0.705	0.352	-17.110	0.3082E+13	7.816	1.306	2.528	0.227
0.4551E+04	0.692	0.346	-16.990	0.3473E+13	7.875	1.274	2.315	0.365
0.5294E+04	0.696	0.348	-17.200	0.3865E+13	7.965	0.783	2.867	0.437
0.4986E+04	0.688	0.344	-17.130	0.4133E+13	7.952	1.238	3.365	0.507
0.8359E+04	0.672	0.336	-17.117	0.5052E+13	8.177	1.196	2.253	0.266
0.7933E+04	0.682	0.341	-17.340	0.5594E+13	8.129	1.197	3.650	0.511

Notes:

Column	Index Number	Appendix A Definition
Col. (10)	Kolm's index	(KOLM)
Col. (11)	Relative mean deviation #1	(RMD1)
Col. (12)	Relative mean deviation #2	(RMD2)
Col. (13)	Theil inequality measure #1	(THEIL1)
Col. (14)	Theil inequality measure #2	(THEIL2)
Col. (15)	Theil inequality measure #3	(THEIL3)
Col. (16)	Standard deviation of log of income	(SDL)
Col. (17)	Log of variance of income	(LV)
Col. (18)	Average rate index	(HIND)